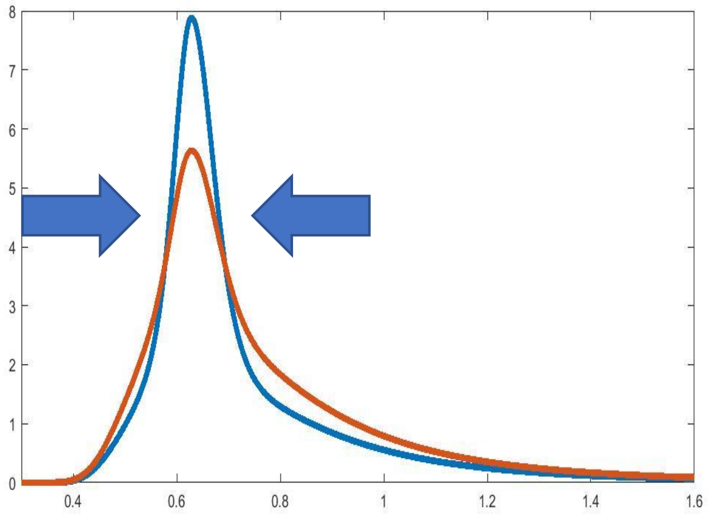
Probability of Occurrence of Extreme Events in Directional JONSWAP Sea States

Cagil Kirezci, Alexander V. Babanin

THE UNIVERSITY OF MELBOURNE

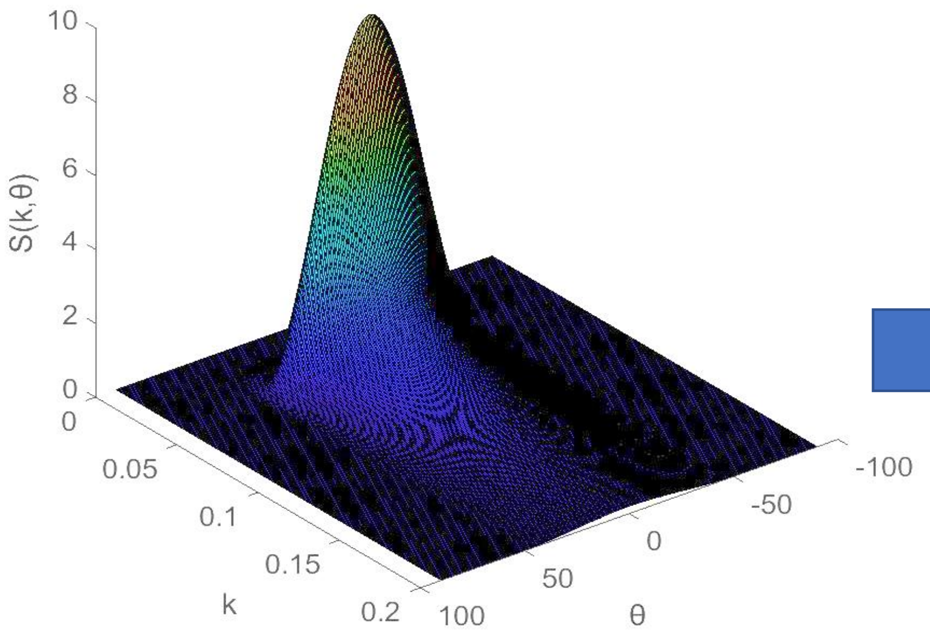
Modulational Instability in Water Waves

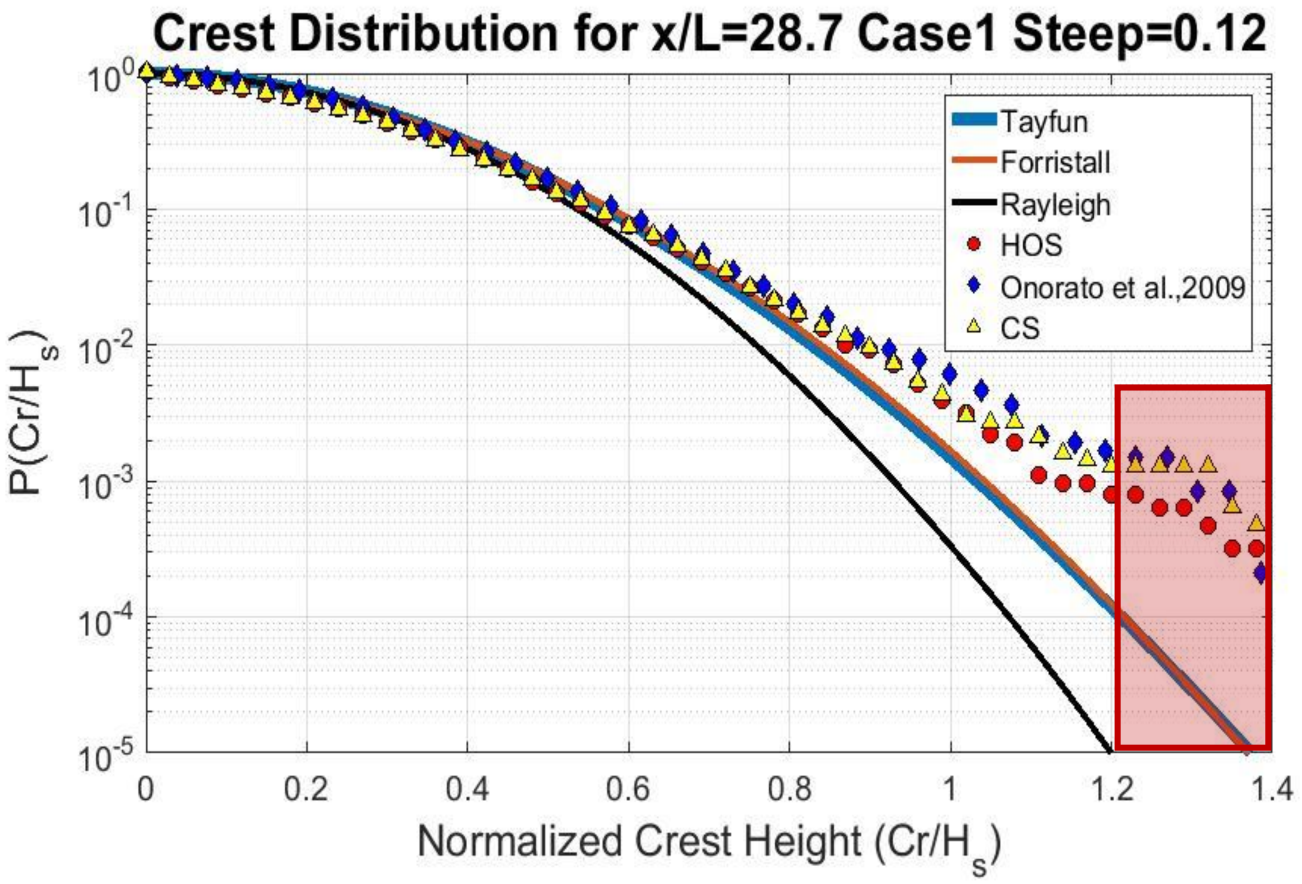
- Modulational instability is defined as the instability of a uniform narrowband wave train to sideband modulations
- Modulational instability is one of the mechanisms that are held responsible for the formation of freak waves.
- If wave steepness is large enough and spectral bandwidth is sufficiently small, modulational instability can also take place in random spectra (Onorato et al., 2006)
- The occurrence frequency of this phenomenon and its initiative conditions in real ocean has still been discussed



Spectrum Analysis for Extreme Events

- By Definition $\frac{H_i}{H_s} > 2\sim2.2\ or\ \frac{C_{max}}{H_s} > 1.2\sim1.3$





Phase Resolving Models

HOS Ocean (Ducrozet, et al.,2016) - High Order Spectral Method

- High Order Spectral Model
- Pseudo-spectral
- Numerical method to approximately solve Euler equations in a rectangular domain with constant depth.
- Two Coordinate system one for surface conditions and one for surface vertical velocity.
- Transition from one coordinate to another is carried out with "Taylor series expansion of the velocity potential at exact surface " (Ducrozet et al.,2016)

$$\phi(x, z, t) = \sum_{m=1}^{M} \phi^{(m)}(x, z, t)$$

where M is the order of approximation in nonlinearity , M=3

Phase Resolving Models

Fully Nonlinear Model (Ch) (Chalikov et al. 2014)

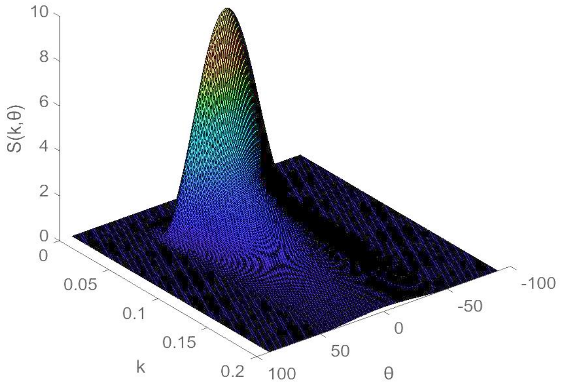
- Based on surface following nonorthogonal curvilinear coordinate system.

$$\xi = x\,, \vartheta = y\,, \tau = t\,, \zeta = z - \eta(\xi, \vartheta, \tau)$$

- Velocity potential is represented as sum of analytical and nonlinear components.
- Kinematic and dynamic conditions on surface considered as evolutionary equations, so that they can be integrated as in 1D model with conformal coordinates.
- Domain is considered as a small part of infinitely large basin.
- Since 3D potential equation turns into elliptical equation to be solved every time step, computational needs increased dramatically.
- Moving periodic wave surface is written as a Fourier series

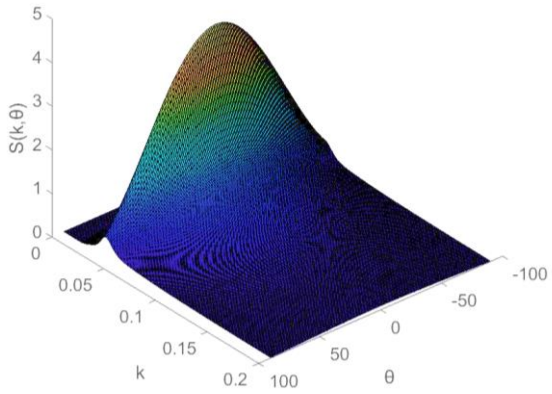
$$\eta(\xi, \vartheta, \tau) = \sum_{-M<k<M} \sum_{-M_y<l<M_y} h_{k,l}\, \theta_{k,l}$$

Spectra



$$S(\omega,\theta) = F(\omega)\,.\,G(\theta)$$

$$F(\omega) = \alpha H_s^2 \omega_p^4 \omega^{-5} \exp\left[-\frac{5}{4}\left(\frac{\omega}{\omega_p}\right)^{-4}\right] \gamma^{\exp\left[-\frac{(\omega-\omega_p)^2}{2\sigma^2\omega_p^2}\right]}$$



A_d= 2.0

$$G(\theta) = \frac{1}{\beta} cos^2\left(\frac{2\pi\theta}{4\beta}\right)$$

Inverse Normalized Directional Spreading → A_d (Babanin and Soloviev, 1998)

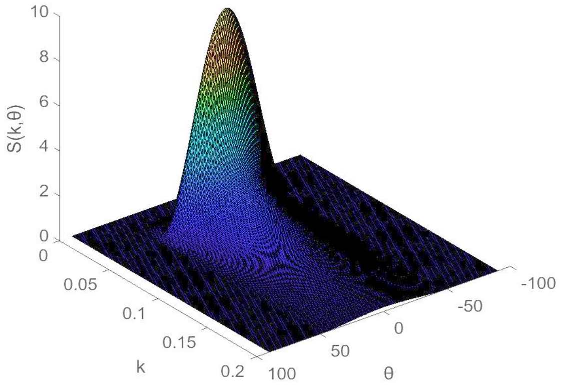
$A_d = 0.7$

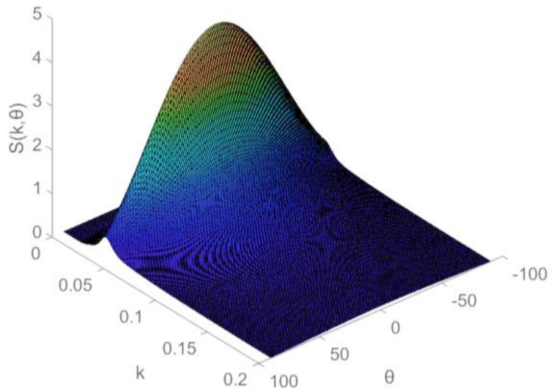
$$\int_{-\pi}^{\pi} \phi(f,\theta)\mathrm{d}\theta = 1, \phi(f,\theta) = A(f)K(f,\theta), \quad A^{-1} = \int_{-\pi}^{\pi} K(f,\theta)\mathrm{d}\theta\,.$$

For wind seas → A_d= 0.8 - 2.0

	α	γ	Ad	Duration	Xlen	Ylen	Resolution	Realization
Ch	*0.006-0.016*	*3.00-6.00*	*0.7-10*	*210Tp*	*40*	*40*	*1024*256*	*5*
HOS-Ocean	*0.006-0.016*	*3.00-6.00*	*0.7-10*	*210Tp*	*40*	*40*	*256*256*	*6*

Spectra





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1092 Spectra

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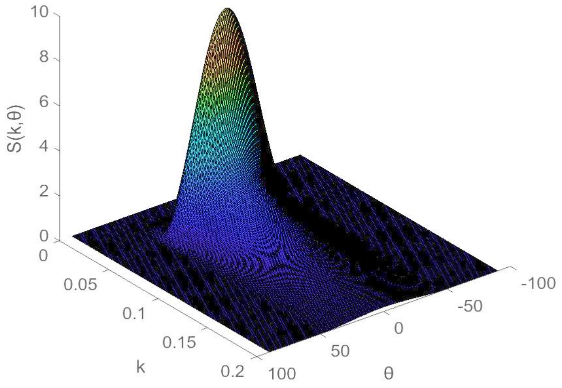

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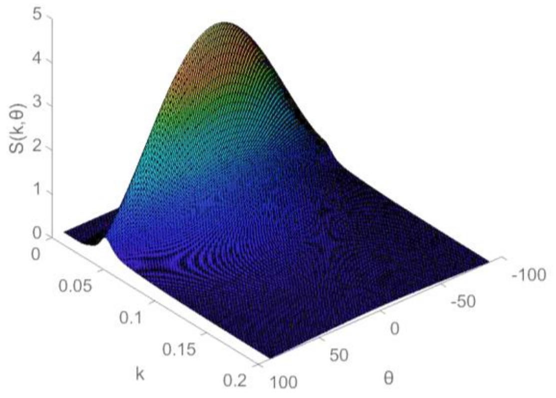

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Over 10000 simulation

Decision Criteria

I. $\mathrm{BFI}_{2\mathrm{D}}$ (Mori et al., 2011)

Directional extend of BFI
Derived based on cubic nonlinear Schrödinger equation simulations with two-dimensional spectrum.

$$BFI_{2D} = \frac{\varepsilon\sqrt{2}}{\sqrt{\delta_\omega^2 + \alpha_3\delta_\theta^2/2}} = \frac{BFI}{\sqrt{1+\alpha R}}$$

II. Π_2 Number (Ribal et al.,2013)

Derived from Alber Equation which describes weakly nonlinear evolution of inhomogeneous wave spectrum.

Π Numbers are based on steepness, JONSWAP parameters α and γ.

Directional bandwidth is represented using A_d, inverse normalized directional spreading depth by Babanin and Soloviev (1998)

$$\Pi_2 = \frac{\varepsilon}{\alpha\gamma} + \frac{\beta}{\varepsilon A_d}$$

III. Directional Modulation Index

(MI_d) (Babanin et al., 2010)

Mid aims to indicate the existence of modulational instability in 2D fields. Based on steepness and A_d

$$MI_d = A_d ako$$

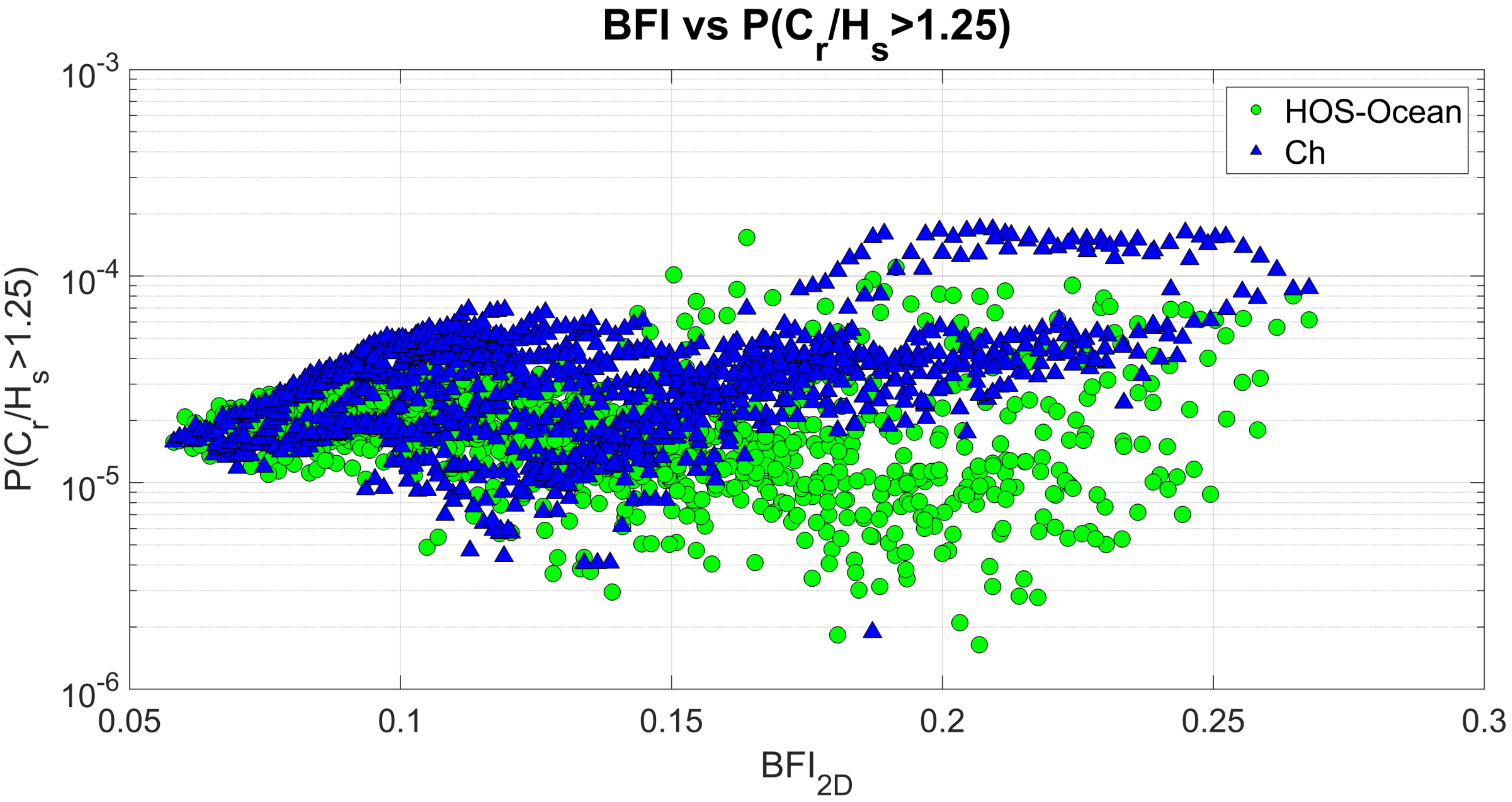
IV. Kurtosis

$$\kappa_4 = 3 + \kappa_4^{(bound)} + \kappa_4^{(dynamic)}$$

$$\kappa_4^{(bound)} = \frac{9}{2} * \varepsilon^2$$ (Janssen, 2009)

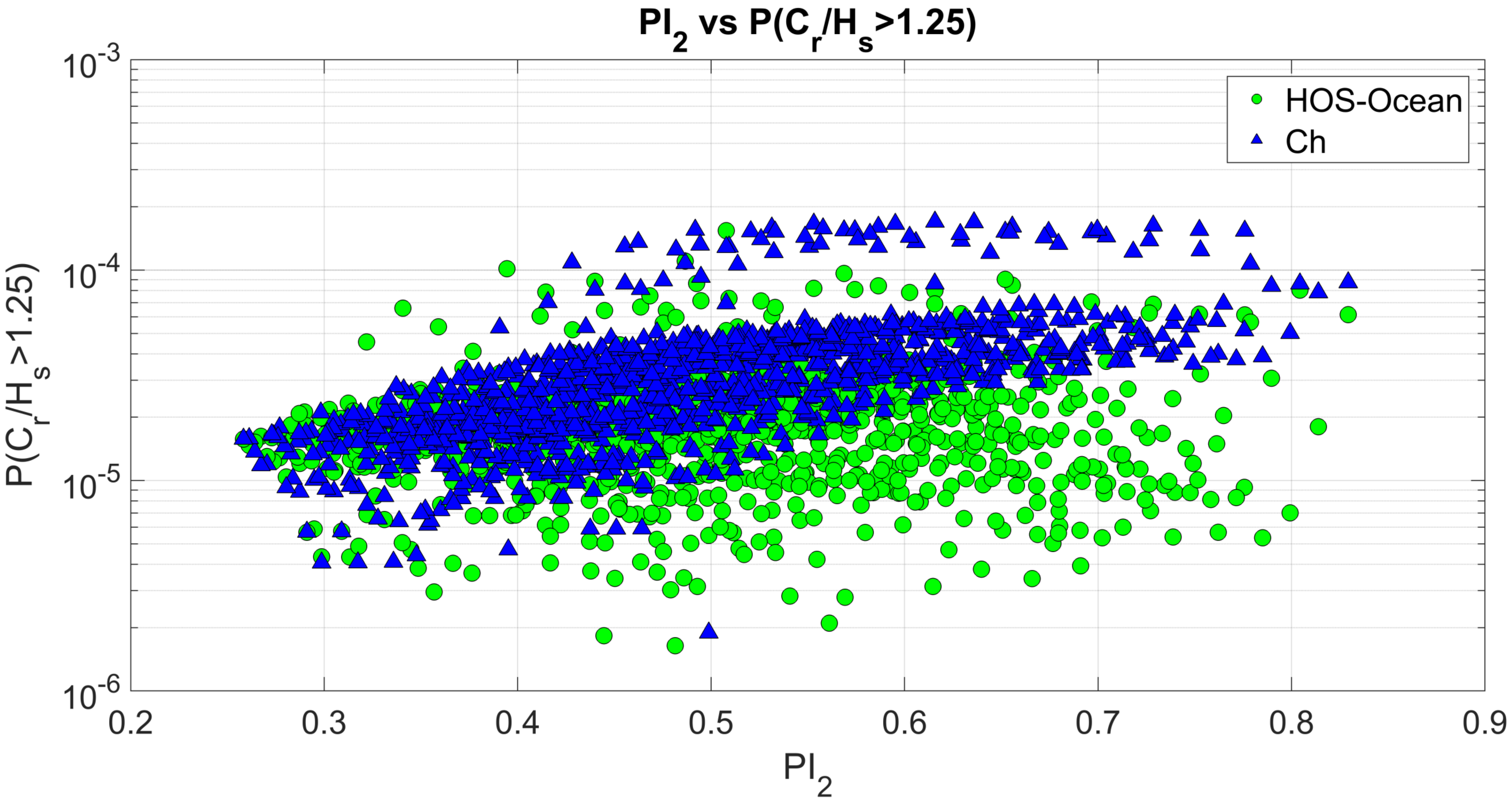
$$\kappa_4^{(dynamic)} = \frac{\pi}{\sqrt{3}} * BFI_{2D}^2$$ (Mori et al.,2011)

BFI_{2D}

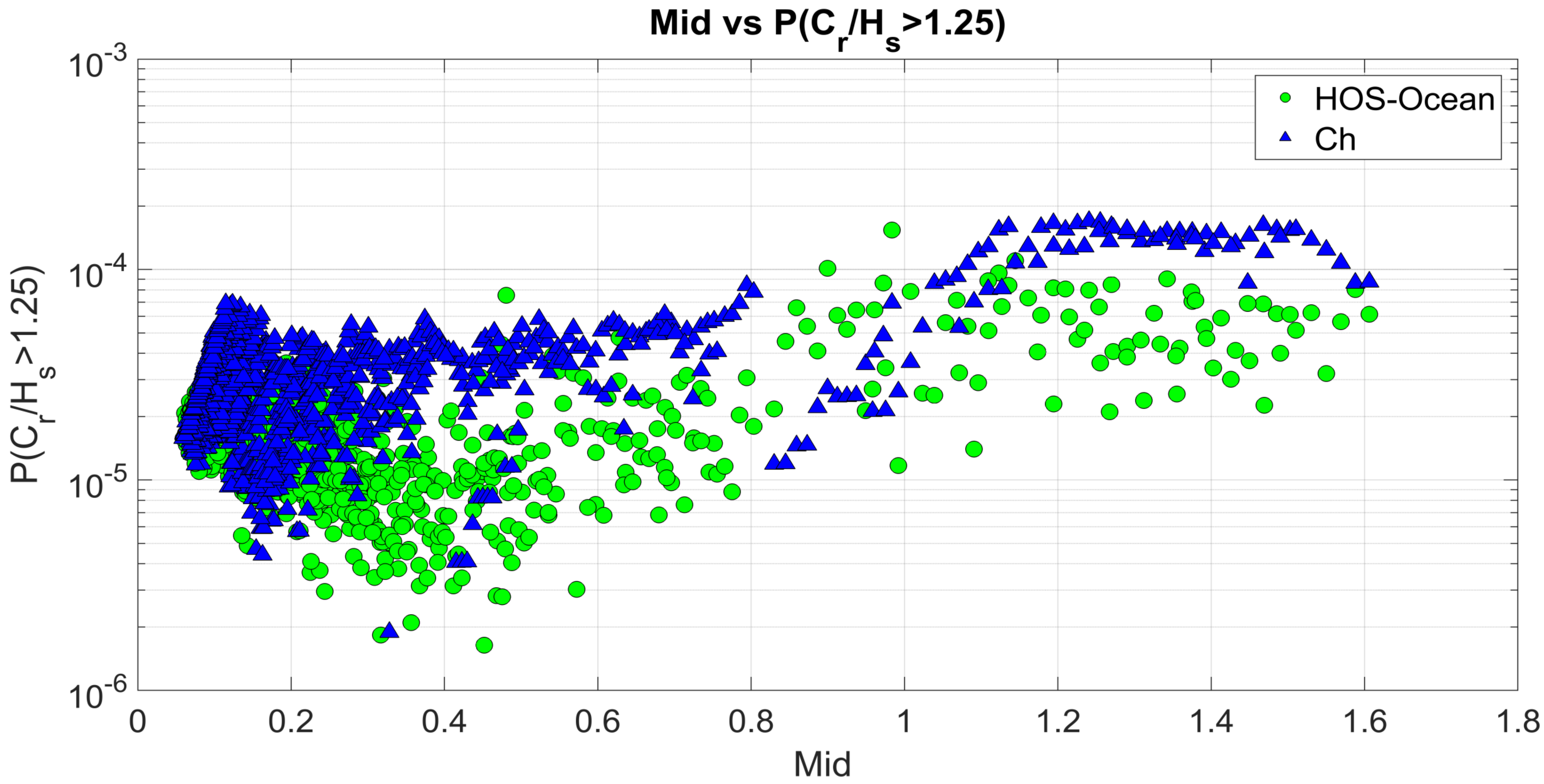


BFI vs P(C_r/H_s>1.25)
10^-3
10^-4
10^-5
10^-6
P(C_r/H_s>1.25)
0.05
0.1
0.15
0.2
0.25
0.3
BFI_2D
HOS-Ocean
Ch

Π_2

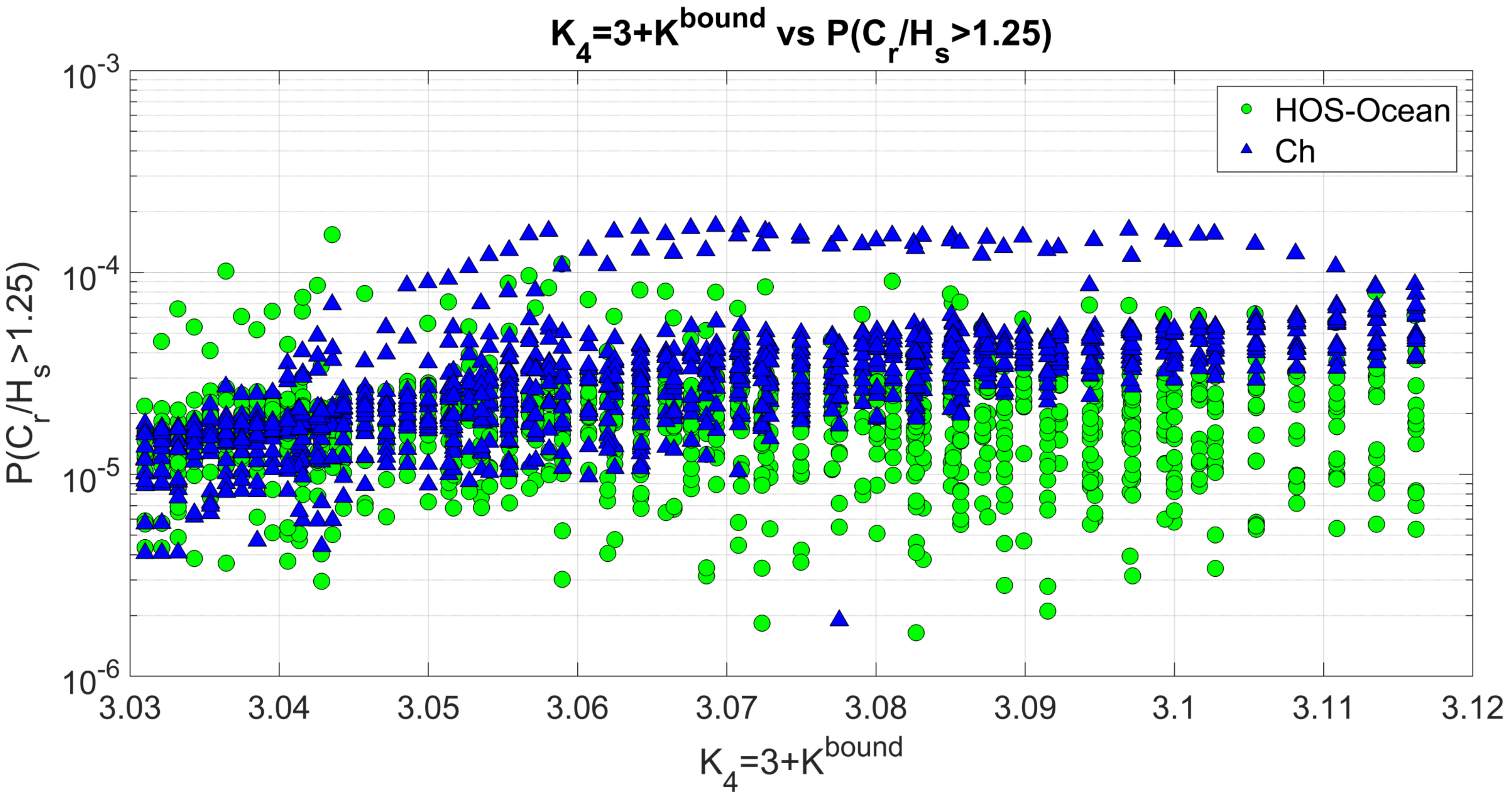

PI2 vs P(Cr/Hs>1.25)
10-3
10-4
10-5
10-6
P(Cr/Hs>1.25)
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
PI2
HOS-Ocean
Ch

$$MI_d = A_d ako$$

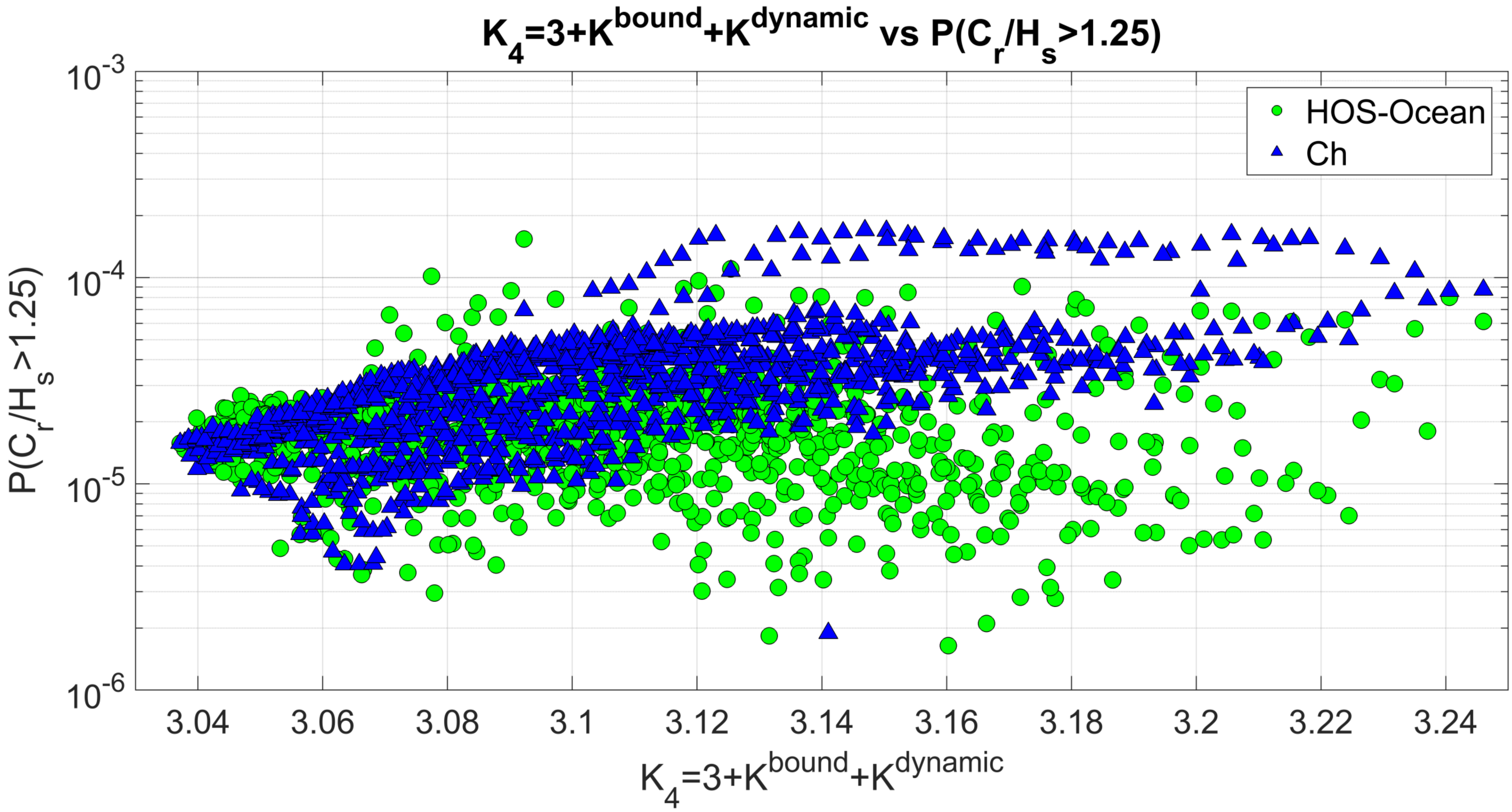


Mid vs P(C_r/H_s>1.25)
10^-3
10^-4
10^-5
10^-6
P(C_r/H_s>1.25)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
Mid
HOS-Ocean
Ch

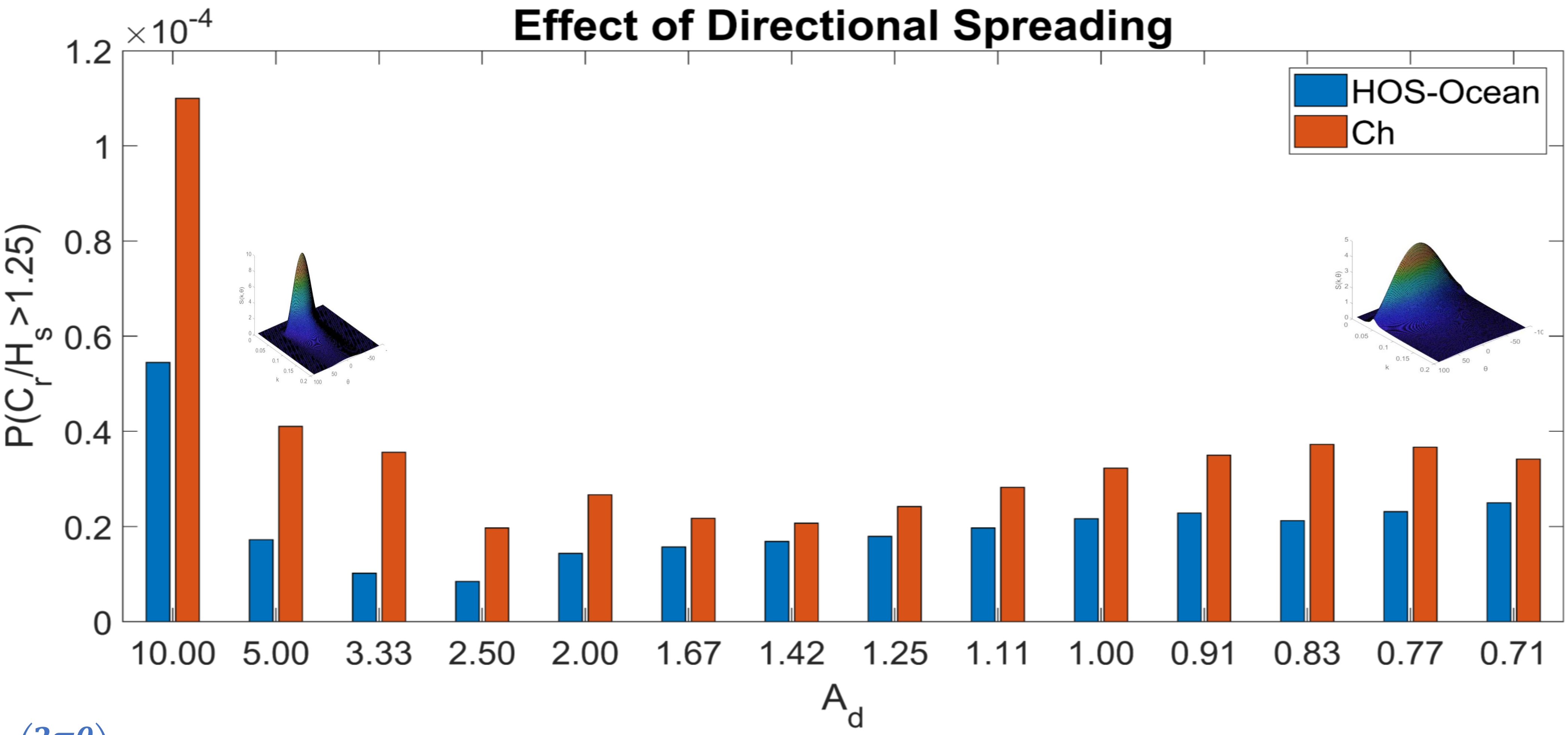
$$\kappa_4 = 3 + \kappa_4^{(bound)}$$



$$\kappa_4 = 3 + \kappa_4^{(bound)} + \kappa_4^{(dynamic)}$$



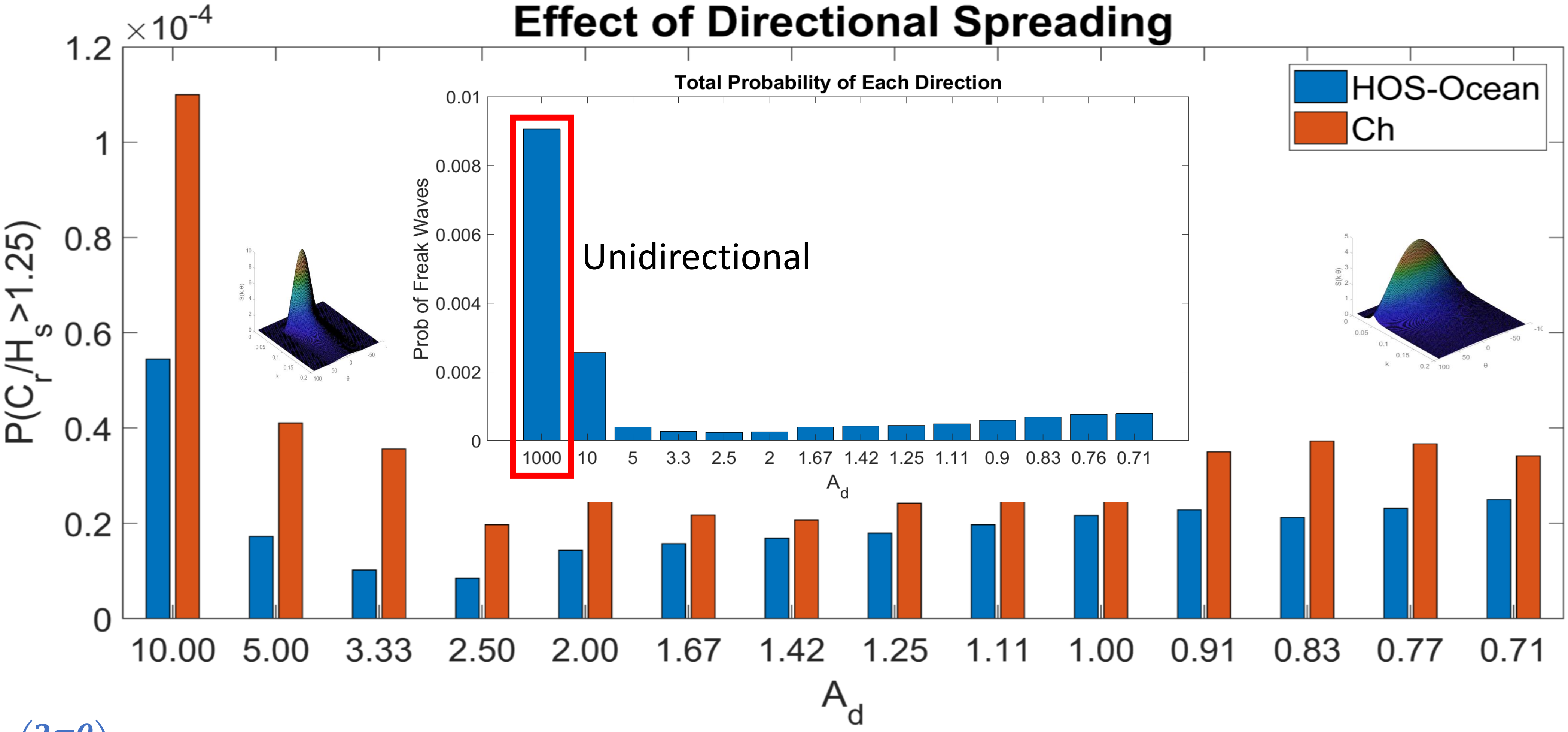
Directional Spreading



$\beta \rightarrow \frac{1}{\beta} cos^2\left(\frac{2\pi\theta}{4\beta}\right)$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
$s \rightarrow cos^{2s}(\theta/2)$				78	50	35	25	19	15	12	10	8	7	6

Directional Spreading Increases

Directional Spreading



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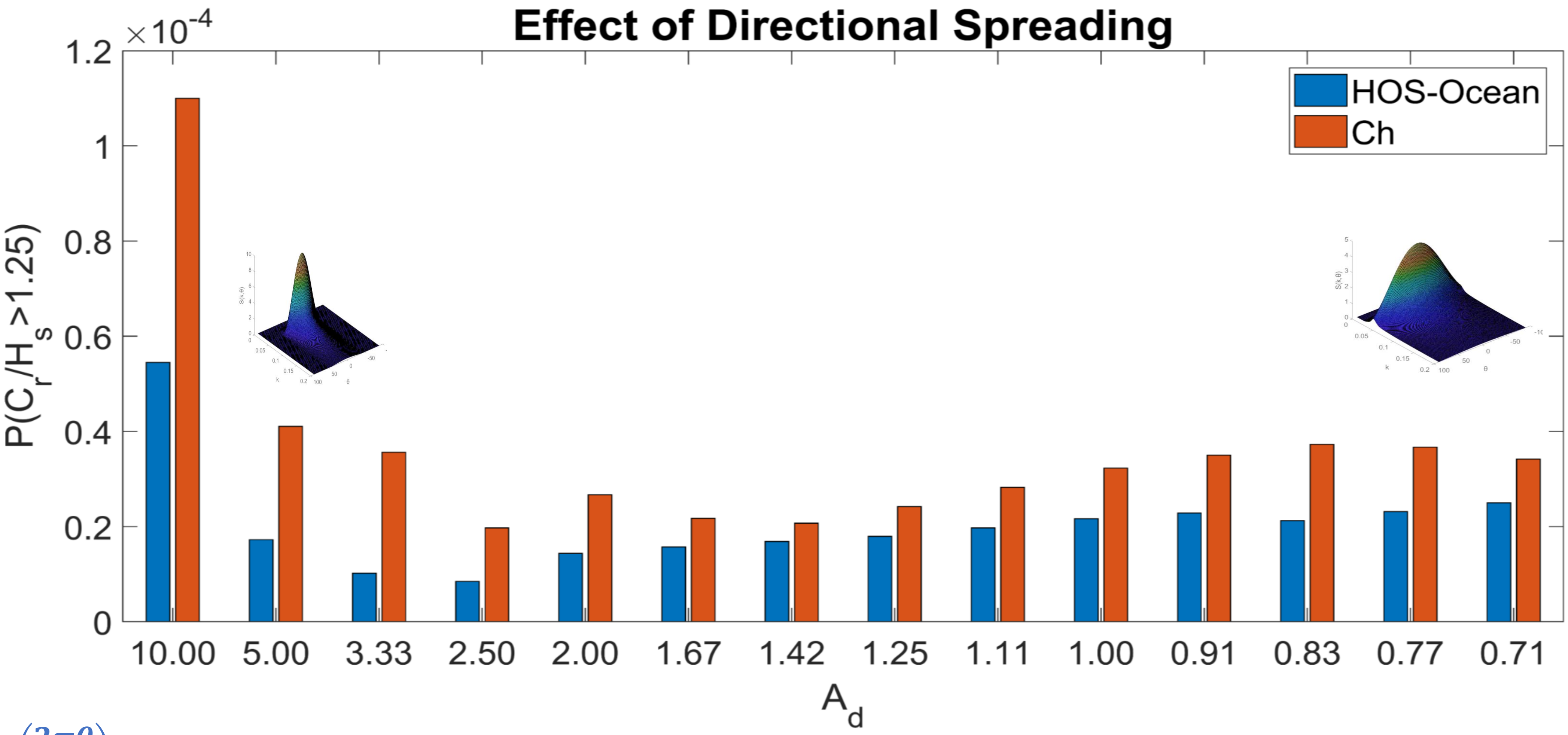
$s \rightarrow cos^{2s}(\theta/2)$ 78 50 35 25 19 15 12 10 8 7 6

Directionality Spreading Increases

Directional Spreading



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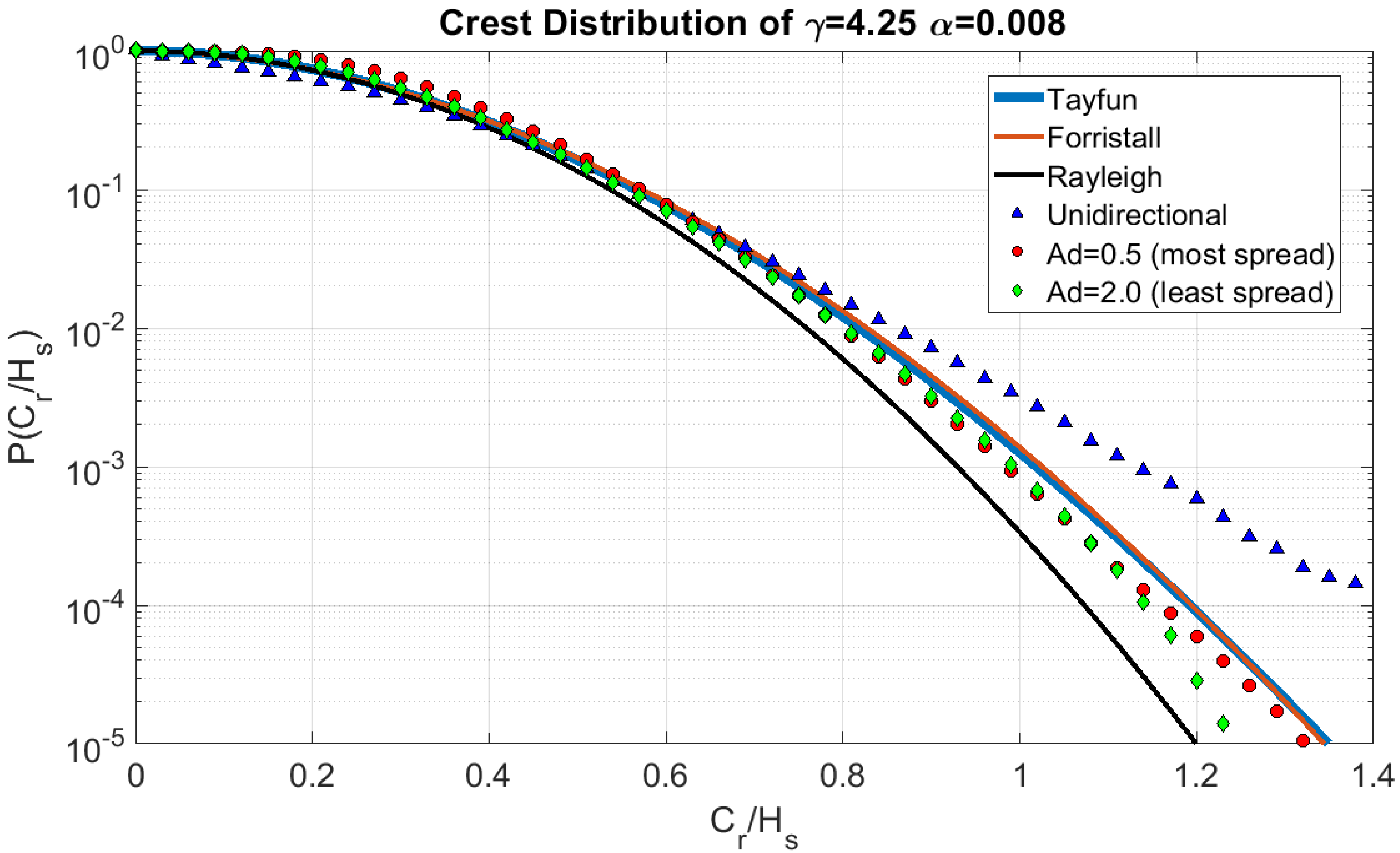
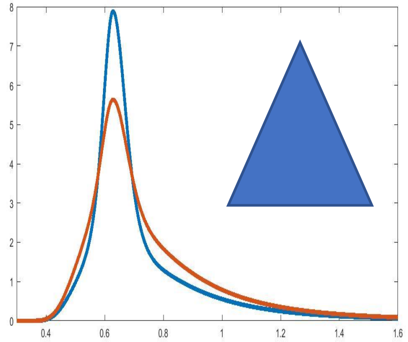
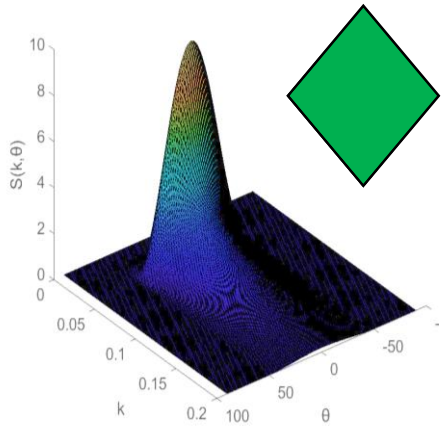
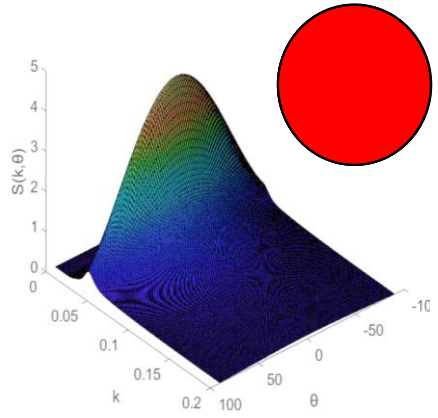
Directionality Spreading Increases

Crest Distribution









Discussion & Conclusion

- ***Limited to JONSWAP sea states***
- ***Observed Freak waves cannot be always associated with initial spectral conditions.***
- Crest distributions usually lie above Rayleigh , close to second order distributions. No strong deviations.
- Occurrence of modulational instability in realistic unimodal spectrum is significantly small. (1e-4 to 1e-5)
- For certain directionality range (highly spread sea states), freak wave occurrence is increased with increasing directionality, which can be only explained by the increase in freak waves formation by spatial focusing (linear mechanism).
- Occurrence of Freak Waves in high directional spreading is very sudden with respect to highly spread sea states.
- Ch model results show higher correlation with decision criteria.

Discussion & Conclusion

- Threshold limits are hard to determine for the considered decision criteria.
- $\boldsymbol{\Pi_2}$ decision criteria shows the higher correlation with freak wave occurrence probability among all decision criteria
- Highest correlation is between kurtosis and freak wave probability is achieved for

$$\boldsymbol{\kappa_4 = 3 + \kappa_4^{(bound)} + \kappa_4^{(dynamic)}}$$

- Correlation between decision criteria($\boldsymbol{\Pi_2}$ and BFI) and higher accuracy of kurtosis definition including which BFI, confirms the relevance of BFI like parameters in for JONSWAP spectra.
- Low correlation of Mid shows the importance of frequency bandwidth on Freak wave probability.
- Computational cost of HOS is less than Ch. Operational usage of both models are still highly challenging.

THANK YOU

Discussion & Conclusion

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- Occurrence of modulational instability in realistic unimodal spectrum is significantly small. (1e-4 to 1e-5)
- For certain directionality range (highly spread sea states), freak wave occurrence is increased with increasing directionality, which can be only explained by the increase in freak waves formation by spatial focusing (linear mechanism).
- Life of Freak Waves are longer when directional spreading is small with respect to highly spread sea states. (Modulational Instability)
- Ch model results show higher correlation with decision criteria.
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